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Nonlinear Dynamical Systems for Malaria Transmission and Control: Ross-Macdonald Model

(A Case Study of Yobe State University Teaching Hospital (YSUTH), Yobe State Specialist Hospital and Potiskum general hospital)

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Abstract: Malaria was declared an emergency in Nigeria and strategies for the control of Malaria in Nigeria were adopted to reduce its prevalence to a level at which the disease will no longer constitute public health problems. In this work, we presented a deterministic (Ross–Macdonald model susceptible, expose/ infected, infectious and recovered) model incorporating the method of control adopted by national Malaria and leprosy control program. We established the disease free and the endemic equilibrium states and carried out the stability analysis of the disease. Free and the equilibrium state. We also carried out numerical simulation of the model to have an insight into the dynamics of the model. We found out that the disease free equilibrium state is stable. The feedback dynamics from mosquito to human and back to mosquito involve considerable time due to the incubation periods of the parasites. In this paper, taking explicit account of the incubation periods of parasites within the human and the mosquito, we first propose a Ross–Macdonald model. The Jacobiant results showed that it would be very difficult to completely eradicate Malaria from Nigeria using the method adopted by national Malaria and leprosy control program.

Keyword: Malaria; Ross–Macdonald; Jacobiant results

Introduction

Malaria is a parasitic disease transmitted by female anopheles' mosquitoes. It is a major cause of death in many developing countries of the world; also consider as one of the most prevalent diseases in tropical and subtropical areas. World Health Organization (WHO) estimated 216 million cases of malaria, 81% in the African region and there were 655,000 malaria deaths, 91% in the African region, and 86% were children under 5 years of age. Malaria is the third leading cause of death for children under five years worldwide, after pneumonia and diarrheal disease [1, 3]. The intensity of transmission depends on factors related to the parasite, the vector, the human host, and the environment.

Transmission is more intense in places where the mosquito lifespan is longer (so that the parasite has time to complete its development inside the mosquito) and where it prefers to bite humans rather than other animals. Transmission also depends on climatic conditions that may affect the number and survival of mosquitoes, such as rainfall patterns, temperature and humidity [4]. The incidence of malaria has been growing recently due to increasing parasite drug-resistance and mosquito insecticide-resistance. Therefore, it is important to understand the important parameters in the transmission of the disease and develop effective solution and strategies for its prevention and control [2].

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Many epidemic models have been analyzed mathematically and applied to specific diseases [3, 5]. Since the first mathematical model of malaria transmission is introduced by Ross [6], quite a few mathematical models have been formulated to investigate the transmission dynamics of malaria [6–13]. Ngwa and Shu [7] analyze a deterministic differential equation model for endemic malaria involving variable human and mosquito populations. We present an ordinary differential equation mathematical model for the spread of malaria in human and mosquito populations. Susceptible humans can be infected when they are bitten by an infectious mosquito. They then progress through the exposed, infectious, and recovered classes, before reentering the susceptible class. Susceptible mosquitoes can become infected when they bite infectious or recovered humans, and once infected they move through the exposed and infectious classes. Both species follow a logistic population model, with humans having immigration and disease-induced death.

In Nigeria, malaria prevalence is as high as 80 to 85% and is the most common cause of outpatient visits to health facilities. The malaria situation in Nigeria is very burdensome and it impedes human development. The degree of malaria infestation varies from region to region in Nigeria. The rate of malaria infection across space depends on dynamic processes involving complex climatic, environmental, physical, and social variables operating differently in space. This complexity makes the analysis of the spatial pattern of malaria infection in Nigeria important [2]. Yobe State is among the Nigerian States suffering from the scourge of malaria all year round.

In this work, the Researchers aimed to study the spread of malaria disease in Yobe State using a nonlinear dynamical system to qualitatively analyze its dynamics. Ross–Macdonald malaria model with mobile human hosts, mobile vectors and heterogeneous environment is considered.

Statement of the Problem

Non Linear Dynamical Systems for Malaria transmission and control: Ross-Mac Donald model ―the spread of Malaria through Vector insect using Ross-MacDonald Model to enable make a prediction or determine the rate at which the disease is spreading.

Research Question

For the fact that Ross-MacDonald Model depict the relationship between parameters and their changes with respect to time hence the need to answer these questions.

- Are there possible outbreak infecting the host community, Damaturu Local Government?
- What is the effect of vaccination in the given community prior to the arrival of diseases?
- What are the epidemic levels?
- How many will get infected if the epidemic takes off?

Research Objectives

To determine the rate at which a Vector insect disease (Malaria) Spread with time and be able to project it impact over a short period of time.

Significant of the Study

The emergence and re-emergence of Malaria Vector insect disease in the temperate state of this country that causes death to millions of children especially those below five (5) years of age is main point of concerned. Hence the need for this research. Significant of the study.

RESEARCH HYPOTHESIS

A Ross-MacDonald Model is a model that predicts the exponential growth rate of malaria in a community of host Damaturu Local Government if no isolation was made especially in a closed population.

Methodology

A Stability Analysis method of solving Ross-MacDonald Model equation is employed completely so that the result obtained could be used as a tool for solving the problem.

Frame Work

This research is mainly the Ross-MacDonald Model equation with much emphasis on the nonlinear system of Stability Analysis.

Literature Review

Model Formulation:

Ross-MacDonald Model:

To study the transmission and spread of malaria in two interacting population of humans (the host) and mosquitoes' H (the vector), we formulate a model which subdivides the total human population size at time (t), denoted by S into susceptible humans exposed humans and recovered humans $R_0(t)$. Hence, we have; Let X(t) denote the numbers of infected humans and Y(t) Proportion of susceptible infections recovered at time t, respectively. Let a be the rate of biting on humans by a single mosquito (number of bites per unit time). Then the number of bites on humans per unit time per human is a/H. If b is the proportion of infected bites on humans that produce an infection, the interaction between the Proportion of susceptible infections recovered Y(t) and the uninfected humans $H - X(t)$ will produce new infected humans of $(a/H)b[H - X(t)]Y(t)$. The incubation period in a human has duration τ_1; it is possible that some individuals recovered from parasitemia during this incubation period (Smith and McKenzie, 2004). Thus, of those individuals infected τ_1 unit times ago, only a proportion $(a/H)b[H - X(t - \tau_1)]Y(t - \tau_1)e^{-r\tau 1}$ is infectious at the present time t, where r is the per capita rate of recovery in humans so that 1/r is the duration of the disease in humans. Therefore, the equation for the rate of change in the number of infected humans

$$\frac{dX}{dt} = -rX(t)\left(\frac{a}{H}\right)b[H(t-\tau_1)]Y(t-\tau_1)e^{-t\tau_1}$$

if μ is the per capita rate of mortality in vectors so that $1/\mu$ is the life expectancy interval in the mosquito has duration τ_2, and c is the transmission efficiency from mosquito to human, then we have the equation for the rate of change in the n of infected human:

$$\frac{dX}{dt} = -rX(t)\left(\frac{a}{H}\right)b[H(t-\tau_1)]Y(t-\tau_1)e^{-t\tau_1}$$

Now define

$$x(t) = \frac{X(t)}{H}, \qquad y(t) = \frac{Y(t)}{M}, \qquad m = \frac{M}{H}.$$

People who have malaria usually suffer chill, fever, sweating, and headache, and may develop severe complications such that they need to seek medical care in hospitals or take breaks at homes. Hospital or home stay plus health education campaign could reduce patient exposure to mosquitoes. In countries approaching malaria elimination,
The stated variables and parameters used for the transmission model are described as the follows;

$x(t)$ Proportion of infected humans

$y(t)$ Proportion of susceptible infections recovered

m ratio of mosquitos to humans
a biting rate on a human per mosquito
b infected mosquito to human transmission efficiency
c infected human to mosquito transmission efficiency
r per capita human recovery rate
μ per capita mortality rate of mosquitos
τ_1 incubation period for *P. vivax* in humans
τ_2 incubation period in mosquitos 5–15 days
$\frac{1}{r}$ average time spend in an infectious state

$$\frac{dx}{dt} = -rX(t) + abm(1 - x(t - \tau_1))y(t - \tau_1)e^{-t\tau_1} \quad (1.0)$$

$$\frac{dy}{dt} = -\mu y(t) + acx(t - \tau_2)(1 - x(t - \tau_2))e^{-\mu\tau_2} \quad (1.1)$$

We have to analyze the existence of equilibria and their stability for model (1).

$$R_0 = \frac{a^2 bcme^{-t\tau_1}e^{-\mu\tau_2}}{r\mu} \quad (2.0)$$

To formulate of variable and parameters,

Let $r = ace^{-\mu\tau_2}$ and $\mu = abcme^{-t\tau_1} + r$ in the first variable and the second variable $r = ace^{-\mu\tau_2} + \mu$ and $\mu = abcme^{-t\tau_1}$ substitute in equation (2.0)

In the first quadrant, system (1) *has at most two equilibria*. *More precisely*,

Stability Analysis

(i) *If R*0 ≤ 1, *then system* (1) *has a unique trivial equilibrium (*0, 0*)*;

(ii) *If R*0 > 1, *then system* (1) *has two equilibria, the trivial equilibrium (*0, 0*) and the postive equilibrium (x*∗*,y*∗*)*, where

$$x = \frac{a^2 bcme^{-t\tau_1}e^{-\mu\tau_2} - r\mu}{ace^{-\mu\tau_2}\left(abcme^{-t\tau_1} + r\right)} = \frac{R_0 - 1}{R_0 + \frac{ace^{-\mu\tau_2}}{\mu}} \quad (3.0)$$

$$y = \frac{a^2 bcme^{-t\tau_1}e^{-\mu\tau_2} - r\mu}{abcme^{-t\tau_1}\left(ace^{-\mu\tau_2} + \mu\right)} = \frac{R_0 - 1}{R_0 + \frac{abcme^{-t\tau_1}}{r}} \quad (3.1)$$

We discuss the stability of *(*0, 0*)* and *(x∗, y∗)*. First we consider the linearized system in equation (1) at *(*0, 0*)*

$$\frac{dx}{dt} = -rx(t) + abmy(t-\tau_1)e^{-t\tau_1} \quad \ldots\ldots(4.0)$$

$$\frac{dy}{dt} = -\mu y(t) + acx(t-\tau_2)e^{-\mu\tau_2} \quad \ldots\ldots(4.1)$$

The characteristic equation associated with system (4.0) and (4.1) takes the form

$$(\lambda+r)(\lambda+\mu) - a^2bcme^{-t\tau_1-\mu\tau_2}e^{-(t\tau_1+\mu\tau_2)\lambda} = 0 \quad \ldots\ldots(5.0)$$

Then from equation (5.0)

Let

$$F(\lambda,\tau_1,\tau_2) = (\lambda+r)(\lambda+\mu) - a^2bcme^{-t\tau_1-\mu\tau_2}e^{-(t\tau_1+\mu\tau_2)\lambda}$$

It is that $F(\lambda,\tau_1,\tau_2)$ is an analytic function. $F(0,\tau_1,\tau_2) = r\mu(1-R_0)$, and $F(\lambda,0,0) = \lambda_2 + (r+\mu) + r\mu - a^2bcm.$ To discuss the distribution of the roots of the transcendental Equation (5.0), we consider three cases.

Case i: If $R_0 < 1$, then $F(0, \tau_1, \tau_2) > 0$ and $F^1{}_\lambda(\lambda, \tau_1, \tau_2) > 0$ for all positive λ, τ_1 and τ_2. Hence, Equation (5.0) has no zero root and positive roots for all positive τ_1 and τ_2. Now we claim that Equation (5.0) does not have any purely imaginary roots. Suppose that Eq. (5.0) has a pair of purely imaginary roots $\pm\omega i$, $\omega>0$ for some τ_1 and τ_2. Then ω must be a positive root of equation below:

$$\omega^4 + (r^2+\mu^2)\omega^2 + r^2\mu^2 - \left(a^2bcme^{-t\tau_1-\mu\tau_2}\right)^2 = 0 \quad \ldots\ldots(6.0)$$

From equation (6.0) does not have nonnegative real roots since $R_0 < 1$. Hence, Equation (5.0) does not have any purely imaginary roots. On the other hand, $F(\lambda, 0, 0) = 0$ has two negative real roots $\lambda\pm$ since $R_0 < 1$,

$$\lambda\pm = \frac{-(r+\mu) \pm \sqrt{(r+\mu)^2 - 4(a^2bcm - r\mu)}}{2}$$

Also $F^1_\lambda(\lambda\pm,0,0) = 0$. By the implicit function theorem and the continuity of $F(\lambda,\tau_1, \tau_2)$, we know that all roots of (5) have negative real parts for positive τ_1 and τ_2, which implies that *(*0, 0*)* is stable.

Case ii: If $R_0 = 1$, then $F(0, \tau_1, \tau_2) = 0$ and $F^1_\lambda(\lambda,\tau_1,\tau_2) > 0$ for $\lambda \geq 0$, $\tau_1 > 0$ and $\tau_2 > 0$.
Hence, Equation (5.0) has a simple zero root and no positive root for all positive τ_1 and τ_2.

Using a similar argument as in (i), we can obtain that except a zero root, all roots of(5)have negative real parts for positive τ_1 and τ_2. Thus, *(*0, 0*)* is a degenerate equilibrium of codimension one and is stable except in one direction.

Case iii: If $R_0 > 1$, then $F(0, \tau_1, \tau_2) < 0$ and $F^1_\lambda(\lambda,\tau_1,\tau_2) > 0$ for $\lambda \geq 0$, $\tau_1 > 0$ and $\tau_2 > 0$.

Hence, Equation (5.0) has a unique positive real root for all positive τ_1 and τ_2.

To determine the unstable manifold of *(*0, 0*)* when $R0 > 1$, we discuss the stability of the other equilibrium *(x∗, y∗)* when $R0 > 1$.

The stability of the trivial equilibrium *(0, 0)* can also be analyzed via the real eigenvalues of its Jacobian matrix by using a theorem on page 92 in Smith (1995).

Consider the linearized system of (1) at *(x∗, y∗)*

$$\frac{dx}{dt} = -rx(t) - abmy^* e^{-r\tau_1} x(t-\tau_1) + abm(1-x^*)e^{-r\tau_1} y(t-\tau_1) \quad \ldots\ldots(7.0)$$

$$\frac{dy}{dt} = -\mu y(t) + ac(1-y^*)e^{-\mu\tau_2} x(t-\tau_2) - acx^* e^{-\mu\tau_2} y(t-\tau_2) \quad \ldots\ldots(7.1)$$

The characteristic equation associated with system (7.0) and (7.1) takes the form

$$(\lambda+r)(\lambda+\mu) + abmy^* e^{-r\tau_1}(\lambda+\mu)e^{-\lambda\tau_1} + x(t-\tau_1) + acx^* e^{-\mu\tau_2}(\lambda+r)e^{-\lambda\tau_2} - a^2 bm(1-x^*-y^*)e^{-r\tau_1-\mu\tau_2} e^{-(\tau_1+\tau_2)\lambda} \quad \ldots\ldots(8.0)$$

Let *G(λ, τ1, τ2)* denote the function on the left-hand side of the last equation. Note that
G(0, τ1, τ2) = rμ(R0 − 1) > 0.

Case (i). When *τ1 = τ2 = 0*, the equilibrium *(x∗, y∗)*

$$(x^*, y^*) = -\left(\frac{R_0 - 1}{R_0 + \frac{ac}{\mu}}, \frac{R_0 - 1}{R_0 + \frac{abm}{r}}\right) \text{ with } R_0 = \frac{a^2 bcm}{r\mu} \quad \ldots\ldots(9.0)$$

The characteristic Equation (8.0) becomes $\lambda^2 + \beta\lambda + \gamma = 0$,
where

$$\beta = \frac{abm(ac+\mu)^2 + ac(abm+r)^2}{(ac+\mu)(abm+r)} > 0 \quad \gamma = r\mu(R_0 - 1) > 0.$$

The above equation has two negative real roots

$$\lambda\pm = \frac{-\beta \pm \sqrt{\beta^2 - 4r}}{2},$$

$$\text{i.e } \lambda_- < \lambda_+ < 0,$$

Case (ii). When *τ1 > 0, τ2 = 0*, the equilibrium *(x∗ y∗)*

$$(x^*, y^*) = -\left(\frac{R_0 - 1}{R_0 + \frac{ac}{\mu}}, \frac{R_0 - 1}{R_0 + \frac{abm}{r}}\right) \quad \text{with} \quad R_0 = \frac{a^2 bcme^{-r\tau_1}}{r\mu} \quad \ldots\ldots(10)$$

The equilibrium *(x∗, y∗)* exists under the assumption *R0 > 1*, that is, $\tau_1 < \tau_1 = 1$

$$\tau_1 < \tau^*_1 = \frac{1}{r} In \frac{a^2 bcm}{r\mu} \quad \ldots\ldots(11)$$

The characteristic Equation (8.0)

$$\lambda^2 + a_1\lambda + a_0 + [b_1(\tau_1)\lambda + b_0(\tau_1)]e^{-\lambda\tau_1} = 0 \quad \ldots\ldots(12)$$

Where

$$a_1 = r + \mu + acx^*, \qquad a_0 = r\mu + acx^*,$$

$$b_1(\tau_1) = abmy^* e^{-r\tau_1}, \quad b_0(\tau_1) = abm\mu y^* e^{-r\tau_1} - a^2 bm(1 - x^* - y^*)e^{-r\tau_1},$$

By the implicit function theorem and the continuity of the right-hand side function, Equation (12) has negative real roots for small $\tau 1$. Now we want to show that Equation (12) has negative real roots for all $\tau 1 \in [0, \tau *_1)$. To do so we show that Equation (12) does not have any purely imaginary roots for all $\tau 1 \in [0, \tau *_1)$.
Suppose Equation (12) has a pair of purely imaginary roots $\pm\omega i$, $\omega > 0$, for some $\tau 1 \in [0, \tau *_1)$. Then ω must satisfy the following system

$$\omega^2 - a_0 = b_1(\tau_1)\omega \sin\omega\tau_1 + b_0(\tau_1)\cos\omega\tau_1,$$

$$a_1\omega = -b_1(\tau_1)\cos\omega\tau_1 + b_0(\tau_1)\omega\sin\omega\tau_1,$$

Thus, ω must be a positive root of

$$\omega^4 + B_1\omega^2 + C_1 = 0, \quad \ldots\ldots(13)$$

Where

$$B_1 = a_1^2 - 2a_0 - b_1^2(\tau_1) \qquad C_1 = a_0^2 - b_0^2(\tau_1)$$

Clearly, Eq. (13) has no positive roots $\omega 2$ if and only if either (i) $C_1 \geq 0$ and $B_1 \geq 0$ or (ii)

$$B_1 = B_1^2 - 4C_1 < 0$$

We have

$$C_1 = r^2\mu^2(R_0 - 1)(1 + R_0(1 - 2x^*))$$

Similarly, after some tedious computations, we have

$$B_1 = \mu^2 + 2ac\mu x^* + a^2c^2x^{*2} + \frac{r^2(1 - 2x^*)}{(1 - x^*)^2}.$$

Note that $B_1 > 0$ and $C_1 > 0$ if $acr + 2r\mu \geq a2bcm$. Therefore, Equation (13) has no positive roots $\omega 2$ if $1 < R0$ and $a2bcm \leq acr + 2r\mu$. Consequently, Equation (12) does not have any purely imaginary roots for all $\tau_1 \in [0, \tau *1)$ so that Equation (12) only has negative real roots for all
$\tau_1 \in [0, \tau *1)$.

Case (iii). For $\tau 1 \in [0, \tau *1)$, consider $\tau 2 > 0$ so that $1 < R_0$ and $a2bcm \leq acr + 2r\mu$ such that the equilibrium $(x*, y*)$ given by (3.0) exists; that is,

$$\tau_2 \leq \tau_2^*(\tau_1) = \frac{1}{\mu} In \frac{a^2bcme^{-r\tau_1}}{r\mu} \quad \ldots\ldots(14)$$

Clearly, the left-hand side of the characteristic Equation (8.0) is analytic in λ and $\tau 1$, $\tau 2$. As $\tau 2$ varies, by Theorem 2.1 of Ruan and Wei (2003), the sum of the multiplicity of zeros of the left hand side of (8) on the open right half-plane can change only if a zero appears on crosses the imaginary axis.

By Case (ii), Equation (8.0) only has negative real roots for $\tau_2 = 0$ and $\tau 1 \in [0, \tau * 1)$. It follows that there is a . $\tau_2(\tau_1)$, $0 <$. $\tau 2(\tau_1) \leq \tau *_2 (\tau_1)$, such that Equation (8.0) only has negative real roots for $\tau_2 <$. $\tau_2(\tau_1)$.

However, using a similar argument as in Case (ii), we know that the Equation (8.0) only has negative real roots for $\tau_1 = 0$ and $\tau_2 \in [0, \tau$

Summarizing the above analysis, we have the following results on the stability of the equilibrium.

Results And Discussion

Variables And Parameter Values

In this section we used the model formulated in equation three to compute the result at various times and the result obtained can be discuss to enable used depict the epidemic or otherwise from equation, using defined parameters a represented in the table 1.0 and table 1.1 shows the values of some parameters collected from general hospital Damaturu Local Government, Yobe State on six sep, 2021. We will vary the key parameters to investigate the impact of vaccination on the transmission dynamics of Malaria using pictorial representation (Graphs obtained from the tables).

Table 0.1 Variables and parameters

Parameters and Variable	**Dependent Variable**	**Values**	**References**
$x(t)$	proportion of infected humans		
$y(t)$	proportion of susceptible infected recovered		
	Parameters and Constants		
m	ratio of mosquitos to humans	2	[1,2]
a	biting rate on a human per mosquito	0.2-0.5/day	[3, 4, 10]
b	infected mosquito to human transmission efficiency	0.5	[5, 6, 10]
c	Infected human to mosquito transmission efficiency	0.5	[5, 6, 10]
r	per capita human recovery rate	0.01-0.05/day	[3, 6, 7, 10]
μ	per capita mortality rate of mosquitos	0.05-0.5/day	[3, 6, 7]
τ_1	incubation period for *P. vivax* in humans	10-100days	[8, 9, 10]
τ_2	incubation period in mosquitos	1-15days	[3, 9]

Table 1.1 the initial values and parameters are presented constant.

Case	m	a	b	c	r	μ	τ_1	τ_2
1	**2**	**0.2**	**0.5**	**0.5**	**0.05**	**0.05**	**15**	**9**
2	**2**	**0.3**	**0.5**	**0.5**	**0.04**	**0.2**	**18**	**12**
3	**2**	**0.4**	**0.5**	**0.5**	**0.03**	**0.3**	**21**	**15**
4	**2**	**0.5**	**0.5**	**0.5**	**0.02**	**0.4**	**24**	**18**
5	**2**	**0.5**	**0.5**	**0.5**	**0.01**	**0.5**	**29**	**21**

Table 1.2.0 to shows the Susceptible Infected, and Infectious, Time Period from 2020-2021

t	0	1	2	3	4	5
x	70,000	90,000	100,000	130,000	170,000	2000,000
y	25,000	54,000	72,000	84,000	93,000	100,000

Table 1.2.1 to shows the Susceptible Infected, and Infectious, Time Period and Parentage.

t	0	1	2	3	4	5
x	70%	90%	100%	130%	170%	200%
y	25%	54%	72%	84%	93%	100%

$$R_0 = \frac{a^2 bcme^{-r\tau_1 - \mu\tau_2}}{r\mu}$$

$$R_0 = \frac{(0.2)^2 (0.5 \times 0.5 \times 2) e^{-(0.05\times 15)-(0.05\times 9)}}{0.05 \times 0.05}$$

$$= \frac{(0.04)(0.5)e^{-1.2}}{0.0025}$$

$$= \frac{0.02 \times 0.3012}{0.0025}$$

$$= \frac{0.00602}{0.0025}$$

$$= 2.4095$$

$R_0 > 1$

Substitute R_0 in equation (3.0) and (3.1) to find Variable of x and y

$$x = \frac{R_0 - 1}{R_0 + \frac{ace^{-\mu\tau_2}}{\mu}}$$

$$x = \frac{2.4096 - 1}{2.4096 + \frac{(0.2 \times 0.5)e^{-(0.05\times 9)}}{0.5}}$$

$$= \frac{1.4096}{2.4096 + \frac{0.1 \times 1.568}{0.5}}$$

$$= \frac{1.4096}{2.4096 + \frac{0.1568}{0.5}}$$

$$= \frac{1.4096}{2.4096 + 0.3136}$$

$$= \frac{1.4096}{2.7232}$$

$$= 0.5176$$

Therefore x= 0.5176

$$y = \frac{R_0 - 1}{R_0 + \frac{abcme^{-t\tau_1}}{r}}$$

$$= \frac{2.4096 - 1}{2.4096 + \frac{(0.2 \times 0.5 \times 0.5 \times 2)e^0}{0.5}}$$

$$= \frac{1.4096}{2.4096 + \frac{0.1 \times 1}{0.5}}$$

$$= \frac{1.4096}{2.4096 + \frac{0.1}{0.5}}$$

$$= \frac{1.4096}{2.4096 + 0.2}$$

$$= \frac{1.4096}{2.6096}$$

$$= 0.5401$$

Therefore, y=0.5401

Substitute the value of x and y in equation (4.0) and equation (4.1)

When t=0 in X axis which are showing Infections

$$X(t) = -rx(t) + abmy(t - \tau_1)e^{-t\tau_1}$$
$$X(0) = -0.5 \times 0.5176(0) + (0.2 \times 0.5 \times 2 \times 0.5401)(0 - 15)e^{-(0 \times 15)}$$
$$X(0) = 0 + 0.10802 \times -15$$
$$= -14.8919$$ in the First Month

When t=1

$$X(1) = -0.5 \times 0.5176(1) + (0.2 \times 0.5 \times 2 \times 0.5401)(1 - 15)e^{-(1 \times 15)}$$
$$X(1) = -0.2588 + 0.10802 \times -14(2.7080)$$

$$X(1) = -0.2588 + (-1.8078)$$
$$= -38.3918$$ in the Second Month

When t=2

$$X(2) = -0.5 \times 0.5176(2) + (0.2 \times 0.5 \times 2 \times 0.5401)(2-15)e^{-(2\times15)}$$
$$X(2) = -0.2588(2) + 0.10802 \times -13(3.4011)$$
$$X(2) = -0.5176 + 0.10802 \times -44.2156$$
$$= -44.625$$ in the third Month

When t=3

$$X(2) = -0.5 \times 0.5176(3) + (0.2 \times 0.5 \times 2 \times 0.5401)(3-15)e^{-(3\times15)}$$
$$X(3) = -0.2588(3) + 0.10802 \times -12(3.8066)$$
$$X(3) = -0.7764 + 0.10802 \times -45.6792$$
$$= -46.3475$$ in the fourth Month

When t=0 in Y axis susceptible

$$Y(t) = -\mu y(t) + acx(t-\tau_2)e^{-\mu\tau_2}$$
$$Y(0) = -0.05 \times 0.5401(0) + (0.2)(0-9)e^{-(0.05\times9)}$$
$$Y(0) = -0.8 \times -0.7985$$
$$= 0.6388$$ in the First Month

When t=1

$$Y(1) = -0.05 \times 0.5401(1) + (0.2)(1-9)e^{-(0.05\times9)}$$
$$Y(1) = -0.0270 + (-1.6) - 0.7985$$
$$Y(1) = -0.0270 + 1.2776$$
$$= 1.2506$$ in the Second Month

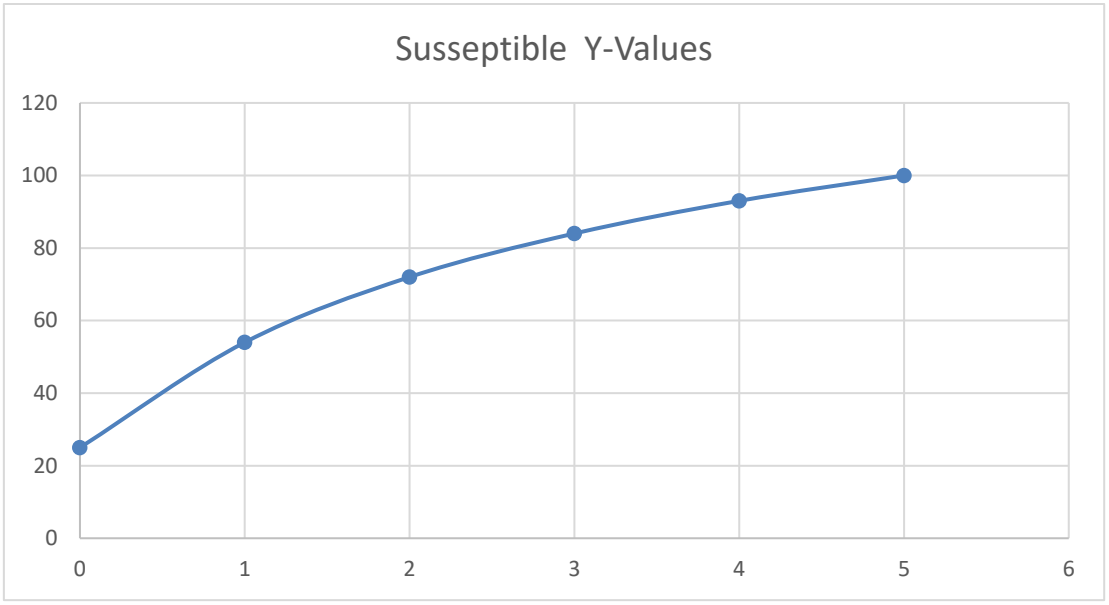
When t=2

$$Y(2) = -0.05 \times 0.5401(2) + (0.2)(2-9)e^{-(0.05\times9)}$$
$$Y(2) = -0.0270(2) + (-1.4) - 0.7985$$
$$Y(2) = -0.054 + 2.1985$$
$$= 2.1445$$ in the Third Month

When t=3

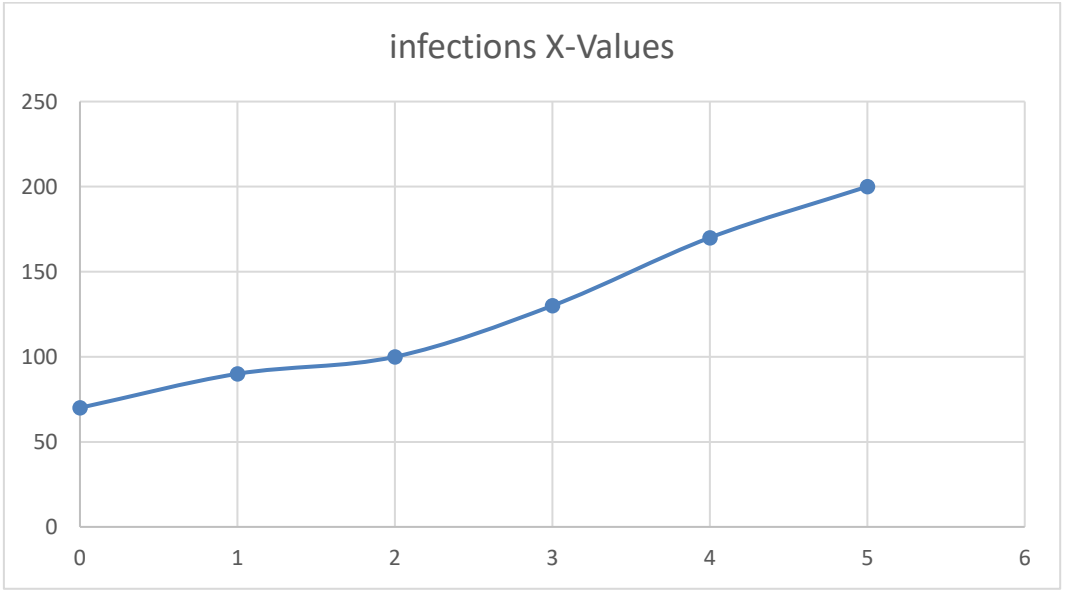
$$Y(3) = -0.05 \times 0.5401(3) + (0.2)(3-9)e^{-(0.05\times9)}$$
$$Y(3) = -0.0270(3) + (-1.2) - 0.7985$$
$$Y(3) = -0.081 + 1.9985$$
$$= 1.9175$$
$$\approx 2.0$$ Approximation in the Fourth Months

Fig.1 Plot for case 1 Susceptible Recovered**,** Time Population Fraction



From **table1**, based on **figure1**, there is a decline in the number of non-infected person in the society of Damaturu Local Government host, although, the number of non-infected decline steadily and latter increase sharply showing in the community.

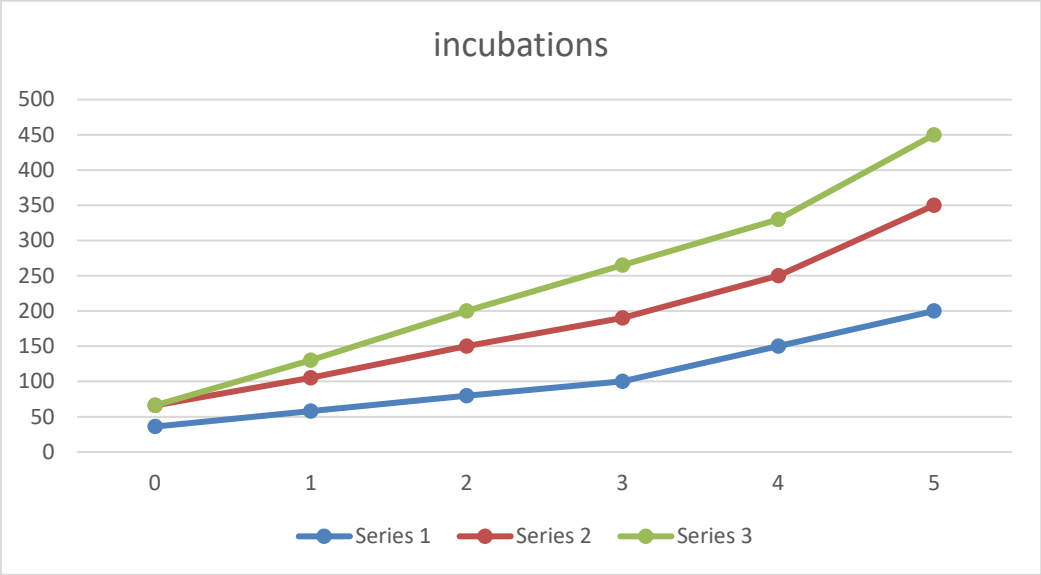
Fig.2 Plot for case 2 Infected Recovered, Time Population Fraction



From **table**, based on **figure2**, there is a decline in the number of infected person in the society of Damaturu Local Government Yobe State host, although, the number of infected decline steadily and later increase sharply showing the community.

Infected 

Fig.3 Plot for case 3 incubations period in a human



From **table 3**, based on **figure 3**, there is a decline in the number of incubation in Human and Mosquitos in the society of Damaturu Local Government Yobe State host, although, the number of incubation in Human and Mosquitos decline steady and latter increase sharply showing a threat to the community.

Time Population Fraction
Incubation of Mosquitos
Incubation of Human
Recovered

However, the **table 3** and **table 2** the above **figure 1** depicts that an epidemic has set up and the level of the epidemic is totally of the final stayed affecting on average of 75,000. People in just four months, that is it infected positive is steady from the onset and letter it exponentially increase causing every large to the society. Also the graph below depicted the non-infected people against the time in the same society of host at that period in time.

Conclusion

Based on the findings so far, Vector insect generates series which converge speedily after some iteration. The epidemic has set up affecting the positive fraction of the community in Damaturu host with as average of 75% infected person pare Month. The epidemic level was slow in the first place but later it increases exponentially indicating a sign of danger to the said community of host.

Recommendation

Sequel to the finding obtained in this study it is obvious that the results obtained have depicted the number of people likely to be infected over a period of time and make a reasonable for best of how many people to be infected in a certain time to enable a proper decision and supply in case of an spread of Vector insect (Malaria) is a contagious bacterial like this in a given society of host with not known in the population number.

Although the study has not make it to our consumption the death that occurred since inception of the epidemic the number of recovered people and the inclining and declining, in the trend of their immune system during Malaria when on treatment. Further study could be conducted by any interested candidate to make clarification of these not known parameter enlisted above by the study.

A study of this nature need data from the host societies which were normally monitored and checked by health agencies difficulties is often encountered while trying to collect the data for the paper. In conclusion I would like to say that since there is an outbreak in the said society of host a special care unit be built by the government in at least every unit of local government so that we can get proper health care.

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